HW #3, 269C, Vese

Due on Friday, May 12 (late homework accepted)

Those that will not turn in the hw #3 in class on Friday, please bring it to my office the following week (under the door if I am not in).

1.

(a) Give a weak variational formulation of the problem

$$\frac{d^4u}{dx^4} = f \quad \text{for} \quad 0 < x < 1,$$

$$u(0) = u''(0) = u'(1) = u'''(1) = 0,$$

and show that the conditions (i)-(iv) are satisfied. Which boundary conditions are essential and which are natural ?

(b) Solve the same problem with the following alternative boundary conditions:

$$u(0) = -u''(0) + \gamma u'(0) = 0, \quad u(1) = u''(1) + \gamma u'(1) = 0,$$

where γ is a positive constant.

2. Give a weak variational formulation of the Robin's problem

$$-\triangle u = f \ \text{ in } \Omega, \quad \gamma u + \frac{\partial u}{\partial n} = g \text{ on } \Gamma,$$

where γ is a constant. When are conditions (i)-(iv) satisfied ?

3. Consider the Neumann problem

$$\begin{aligned} -\triangle u &= f \text{ in } \Omega, \\ \frac{\partial u}{\partial n} &= g \text{ on } \Gamma, \\ \int_\Omega u(x)dx &= 0. \end{aligned}$$

(a) Why condition "$\int_\Omega u(x)dx = 0$" was added here ?

(b) Give a variational formulation of the problem, and prove that the conditions (i)-(iv) are satisfied, under the "usual" assumptions on f and g.

Note: Additional practice problems will be posted on the class webpage.