HW #5, 269C, Vese

Due Wednesday, June 9

1. Consider the PDE (in distributional sense)

$$-\triangle u + k^2 u = f \quad \text{in } R^n.$$

Let $s \in R$. Show that, for all $f \in H^s(R^n)$, there exists a unique $u \in H^{s+2}(R^n)$, solution of the PDE, with $k \in R$, $k \neq 0$.

Hint: use the Fourier transform.

2. Let $I = [0, h]$ and let $\pi v \in P_1(I)$ be the linear interpolant that agrees with $v \in C^2(I)$ at the end points of I. Using the technique of the proof of Thm. 4.1, prove estimates for $\|v - \pi v\|_{L^\infty(\Omega)}$ and $\|v' - (\pi v)'\|_{L^\infty(\Omega)}$, cf. (1.12) and (1.13).

3. Using the results from the textbook, estimate the error $\|u - u_h\|_{H^2(I)}$ for Problem 1.5 and Example 2.4.

Read Section 5.3 (The Plate problem).