

## HW #4, 269C, Vese

Due Friday, May 28, 2004

1. Let  $K$  be a tetrahedron with vertices  $a^i$ ,  $i = 1, \dots, 4$ , and let  $a^{ij}$  denote the midpoint on the straight line  $a^i a^j$ ,  $i < j$ . Show that a function  $v \in P_2(K)$  is uniquely determined by the degrees of freedom:  $v(a^i)$ ,  $v(a^{ij})$ ,  $i, j = 1, \dots, 4$ ,  $i < j$ . Show that the corresponding finite element space  $V_h$  satisfies  $V_h \subset C^0(\Omega)$ .

2. Let  $K$  be a triangle with vertices  $a^i$ ,  $i = 1, 2, 3$ . Suppose that  $v \in P_r(K)$  and that  $v$  vanishes on the side  $a^2 a^3$ . Prove that  $v$  has the form

$$v(x) = \lambda_1(x) w_{r-1}(x), \quad x \in K,$$

where  $w_{r-1} \in P_{r-1}(K)$ .

3. Let  $K$  be a triangle with vertices  $a^i$ ,  $i = 1, 2, 3$ , and let  $a^{ij}$ ,  $i < j$ , denote the midpoints of the sides of  $K$ . Let  $a^{123}$  denote the center of gravity of  $K$ . Prove that  $v \in P_4(K)$  is uniquely determined by the following degrees of freedom

$$\begin{aligned} &v(a^i), \\ &\frac{\partial v}{\partial x_j}(a^i), \quad i = 1, 2, 3, \quad j = 1, 2, \\ &v(a^{ij}), \quad i, j = 1, 2, 3, \quad i < j, \\ &v(a^{123}), \quad \frac{\partial v}{\partial x_j}(a^{123}), \quad j = 1, 2, \end{aligned}$$

Also show that the functions in the corresponding finite element  $V_h$  are continuous.