**Math 269B** Instructor: Luminita Vese. Teaching Assistant: Michael Puthawala.
**Homework #7** Due on: Friday, March 10, or the following Monday.

[**1**] Consider the equation $u_t + a(x)u_x = 0$ for $x \in [0,1]$, $t \in [0,T]$ with $u$ and $a$ periodic in $x$ with period 1. Assume $a(x) \geq 0$ and is continuous for $x \in [0,1]$. Find the value for $\lambda$ for which the scheme (forward-time, backward-space)

$$\frac{v_m^{n+1} - v_m^n}{k} = -a_m \frac{v_m^n - v_{m-1}^n}{h}$$

is stable in the infinity norm $\|\cdot\|_\infty$, under the condition $\frac{k}{h} = \lambda$. Here $a_m = a(x_m)$.

[**2**] Consider the equation $u_t + a(x)u_x = 0$ for $x \in [0,1]$, $t \in [0,T]$ with $u$ and $a$ periodic in $x$ with period 1. Assume $a(x)$ is continuous for $x \in [0,1]$. Find the value for $\lambda$ for which the upwind scheme

$$\frac{v_m^{n+1} - v_m^n}{k} = \begin{cases} -a_m \frac{v_m^n - v_{m-1}^n}{h} & \text{if } a_m \geq 0 \\ -a_m \frac{v_{m+1}^n - v_m^n}{h} & \text{if } a_m < 0 \end{cases}$$

is stable in the infinity norm $\|\cdot\|_\infty$, under the condition $\frac{k}{h} = \lambda$. Here $a_m = a(x_m)$.

[**3**] Give a derivation, based on the Lax-Wendroff idea, of a finite difference method to create approximate solutions of the differential equation

$$u_t + a(x)u_x = 0.$$

What is the leading term of the local truncation error for the scheme you derived ?

[**4**] Consider the modified Lax-Friedrichs scheme for the one-way wave equation $u_t + au_x = f(x,t)$,

$$u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{a\lambda}{1 + (a\lambda)^2}(u_{j+1}^n - u_{j-1}^n) + \triangle t f_j^n.$$

(a) Analize the consistency of the scheme.

(b) Show that this explict scheme is stable for all values of $\lambda$. Discuss the relation of this explicit and unconditionally stable scheme with the Courant-Friedrichs-Lewy Theorem.

CFL Thm: There are no explicit, unconditionally stable, consistent finite difference schemes for hyperbolic systems of partial differential equations.

[**5**] Computational Exercise 6.3.10 from Strikwerda (page 156). You need a routine to solve the implicit equations (see Thomas Algorithm, Section 3.5).