Math 269B, Hw #2

Due on Friday, January 27.

[**1**] (*computational assignment*) Solve problem [1] from the previous assignment, by the leapfrog scheme instead of the Lax-Friedrichs scheme (use a one-step scheme of your choice to compute the initial values at time level $n = 1$).

[**2**] Show that the scheme

$$\frac{u_m^{n+1} - u_m^n}{k} + a\frac{u_{m+2}^n - 3u_{m+1}^n + 3u_m^n - u_{m-1}^n}{h^3} = f_m^n$$

is consistent with the equation $u_t + au_{xxx} = f$ and, if $\nu = \frac{k}{h^3}$ is constant, then it is stable when $0 \leq a\nu \leq \frac{1}{4}$.

[**3**] Show that schemes of the form

$$\alpha v_{m+1}^{n+1} + \beta v_{m-1}^{n+1} = v_m^n$$

are stable if $\Big||\alpha| - |\beta|\Big| \geq 1$. Conclude that the reverse Lax-Friedrichs scheme,

$$\frac{\frac{1}{2}(v_{m+1}^{n+1} + v_{m-1}^{n+1}) - v_m^n}{k} + a\frac{v_{m+1}^{n+1} - v_{m-1}^{n+1}}{2h} = 0$$

is stable if $|a\lambda| \geq 1$.

[**4**] Show that the backward-time central-space scheme is unconditionally stable (use von Neumann analysis).