Math 269B, Hw #4
Due date: Monday, February 7, 2005

[**1**] (*computational assignment*) For $x \in [-1, 3]$ and $t \in [0, 2.4]$, solve the one-way wave equation

$$u_t + u_x = 0,$$

with the initial data

$$u(0,x) = \begin{cases} \cos^2 \pi x & \text{if } |x| \leq \frac{1}{2}, \\ 0 & \text{otherwise }, \end{cases}$$

and the boundary data $u(t, -1) = 0$.

Use the Lax-Friedrichs scheme with $\lambda = 0.8$ and $\lambda = 1.6$ and $h = 1/10$, $h = 1/20$, where $\lambda = \frac{\triangle t}{h}$. At the right boundary use the condition $v_M^{n+1} = v_{M-1}^{n+1}$, where $x_M = 3$.

The Lax-Friedrichs scheme for $u_t + au_x = 0$ is

$$\frac{u_j^{n+1} - \frac{1}{2}(u_{j+1}^n + u_{j-1}^n)}{\triangle t} + a\frac{u_{j+1}^n - u_{j-1}^n}{2h} = 0.$$

Graph or plot solutions at the last time they were computed. What can you say about the behavior of the numerical solutions as a function of h or λ ?

You should turn in the code in the language of your choice, the plots and your interpretation of the tests performed.

[**2**] Show that schemes of the form

$$u_j^{n+1} = \alpha u_{j+1}^n + \beta u_{j-1}^n$$

are stable if $|\alpha| + |\beta| \leq 1$. Conclude that the Lax-Friedrichs scheme is stable if $|a\lambda| \leq 1$.

[**3**] Show that schemes of the form

$$\alpha u_{j+1}^{n+1} + \beta u_{j-1}^{n+1} = u_j^n$$

are stable if $\Big||\alpha| - |\beta|\Big| \geq 1$. Conclude that the reverse Lax-Friedrichs scheme,

$$\frac{\frac{1}{2}(u_{j+1}^{n+1} + u_{j-1}^{n+1}) - u_j^n}{\triangle k} + a\frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2h} = 0$$

is stable if $|a\lambda| \geq 1$.

[4] Show that the following scheme is consistent with the equation $u_t + cu_{tx} + au_x = f$

$$\frac{u_j^{n+1} - u_j^n}{\triangle t} + c\frac{u_{j+1}^{n+1} - u_{j-1}^{n+1} - u_{j+1}^n + u_{j-1}^n}{2\triangle th} + a\frac{u_{j+1}^n - u_{j-1}^n}{2h} = f_j^n.$$