269B Winter 2005, Vese
Homework #1
Due on Friday, January 14

[1] Consider the model problem $u_t = u_{xx}$ for $t > 0$, $0 < x < 1$, with the B.C. $u(0,t) = u(1,t) = 0$ for $t > 0$, and with the I.C. $u(x,0) = u^0(x)$, for $0 \le x \le 1$. Assume that

$$u^0(x) = \begin{cases} 2x, & \text{if } 0 \le x \le \frac{1}{2} \\ 2 - 2x, & \text{if } \frac{1}{2} \le x \le 1. \end{cases}$$

(a) Using separation of variables, obtain an exact analytic solution of the problem (as in Section 2.3, from Morton and Mayers). A few first terms of the series solution can be used to approximate very well the exact solution for small values of t.

(b) Using $\triangle x = 0.05$ and for two different time steps $\triangle t = 0.0012$ and $\triangle t = 0.0013$, compute a numerical approximation using the explicit scheme (forward difference in time, and central second order difference in space). Implement the algorithm and plot two sets of results for each time step at $t = 0$ and after 10, 25 and 50 time steps. Compare the numerical approximations with the exact solution (as in Morton and Mayers, Fig. 2.2) You should turn in the numerical scheme, the code and plots of results.

What can you observe ?