Math 269B, Winter 2003
Instructor: Luminita Vese

Homework #3
Due date: Friday, January 31st.

Note: I strongly encourage you to solve as many exercises as you can from the textbook, and not only those assigned. I also encourage you to work in group.

[1] (*computational assignment*) Solve the one-way wave equation with variable coefficients

$$u_t + (1 + \alpha x)u_x = 0$$

on the interval $[-3, 3]$ and $0 \le t \le 2$ with the Lax-Friedrichs scheme

$$v_m^{n+1} = \frac{1}{2}(v_{m+1}^n + v_{m-1}^n) - \frac{1}{2}a(t_n, x_m)\lambda(v_{m+1}^n - v_{m-1}^n),$$

where $a(t, x) = (1 + \alpha x)$. Consider $\alpha = -0.5$, $\lambda = 1$. Demonstrate that the instability phenomena occur where $|(1 + \alpha x_m)| > 1$. Use the initial data

$$u_0(x) = \begin{cases} 1 - |x| & \text{if } |x| \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Specify the solution to be 0 at both boundaries. You can use a grid spacing of 0.1 ($h = 0.1$).

[2] Show that the scheme

$$\frac{u_m^{n+1} - u_m^n}{k} + a\frac{u_{m+2}^n - 3u_{m+1}^n + 3u_m^n - u_{m-1}^n}{h^3} = f_m^n$$

is consistent with the equation $u_t + au_{xxx} = f$ and, if $\nu = \frac{k}{h^3}$ is constant, then it is stable when $0 \le a\nu \le \frac{1}{4}$.