Math 269B, Hw #2
Due date: Monday, January 27.

Note: I strongly encourage you to solve as many exercises as you can from the textbook, and not only those assigned. I also encourage you to work in group.

[1] (computational assignment) Solve problem [1] from the previous assignment, by the leapfrog scheme instead of the Lax-Friedrichs scheme (use a one-step scheme of your choice to compute the initial values at time level \( n = 1 \)).

[2] Show that the following scheme is consistent with the equation

\[
\frac{u^{n+1}_m - u^n_m}{k} + c \frac{u^{n+1}_{m+1} - u^{n+1}_{m-1} - u^n_{m+1} + u^n_{m-1}}{2kh} + a \frac{v^{n+1}_m - v^n_m}{2h} = f^n_m.
\]

[3] Show that schemes of the form

\[
\alpha u^{n+1}_m + \beta v^{n+1}_m = v^n_m
\]

are stable if \(|\alpha| - |\beta| \geq 1\). Conclude that the reverse Lax-Friedrichs scheme,

\[
\frac{1}{2}(u^{n+1}_{m+1} + u^{n+1}_{m-1}) - \frac{u^n_m}{k} + \frac{a v^{n+1}_m - v^{n+1}_{m-1}}{2h} = 0
\]

is stable if \(|a\lambda| \geq 1\).

[4] Show that the backward-time central-space scheme is unconditionally stable (use von Neumann analysis).