Math 269B, Hw #2
Due date: Monday, January 27.

Note: I strongly encourage you to solve as many exercises as you can from the textbook, and not only those assigned. I also encourage you to work in group.

[1] (*computational assignment*) Solve problem [1] from the previous assignment, by the leapfrog scheme instead of the Lax-Friedrichs scheme (use a one-step scheme of your choice to compute the initial values at time level $n = 1$).

[2] Show that the following scheme is consistent with the equation $u_t + cu_{tx} + au_x = f$

$$\frac{v_m^{n+1} - v_m^n}{k} + c\frac{v_{m+1}^{n+1} - v_{m-1}^{n+1} - v_{m+1}^n + v_{m-1}^n}{2kh} + a\frac{v_{m+1}^n - v_{m-1}^n}{2h} = f_m^n.$$

[3] Show that schemes of the form

$$\alpha v_{m+1}^{n+1} + \beta v_{m-1}^{n+1} = v_m^n$$

are stable if $\Big||\alpha| - |\beta|\Big| \geq 1$. Conclude that the reverse Lax-Friedrichs scheme,

$$\frac{\frac{1}{2}(v_{m+1}^{n+1} + v_{m-1}^{n+1}) - v_m^n}{k} + a\frac{v_{m+1}^{n+1} - v_{m-1}^{n+1}}{2h} = 0$$

is stable if $|a\lambda| \geq 1$.

[4] Show that the backward-time central-space scheme is unconditionally stable (use von Neumann analysis).