Math 269B, Hw #1
Due date: Friday, January 17.

Note: I strongly encourage you to solve as many exercises as you can from the textbook, and not only those assigned. I also encourage you to work in group.

[**1**] (*computational assignment*) For $x \in [-1, 3]$ and $t \in [0, 2.4]$, solve the one-way wave equation

$$u_t + u_x = 0,$$

with the initial data

$$u(0, x) = \begin{cases} \cos^2 \pi x & \text{if } |x| \leq \frac{1}{2}, \\ 0 & \text{otherwise} \end{cases},$$

and the boundary data $u(t, -1) = 0$.

Use the Lax-Friedrichs scheme with $\lambda = 0.8$ and $\lambda = 1.6$ and $h = 1/10$, $h = 1/20$. At the right boundary use the condition $v_M^{n+1} = v_{M-1}^{n+1}$, where $x_M = 3$.

The Lax-Friedrichs scheme for $u_t + au_x = 0$ is

$$\frac{v_m^{n+1} - \frac{1}{2}(v_{m+1}^n + v_{m-1}^n)}{k} + a\frac{v_{m+1}^n - v_{m-1}^n}{2h} = 0.$$

Graph or plot solutions at the last time they were computed. What can you say about the behavior of the numerical solutions as a function of h or λ ?

You should turn in the code in the language of your choice, the plots and your interpretation of the tests performed.

[**2**] Show that the forward-time central-space scheme is consistent with the equation $u_t + au_x = 0$.

[**3**] Show that schemes of the form

$$v_m^{n+1} = \alpha v_{m+1}^n + \beta v_{m-1}^n$$

are stable if $|\alpha| + |\beta| \leq 1$. Conclude that the Lax-Friedrichs scheme is stable if $|a\lambda| \leq 1$.