Math 269 A
HW # 5 Due on Friday, November 16

[**1**] Consider the two stage Runge-Kutta method for solving $y' = f(y)$,

$$\begin{aligned} y^* &= y_i + ahf(y_i) \\ y_{i+1} &= y_i + h(b_1 f(y_i) + b_2 f(y^*)). \end{aligned}$$

(a) Find the coefficients a, b_1 and b_2 so that the method is of 2nd order.
(b) Find the interval of absolute stability for the method.
(c) Does the interval of absolute stability depend upon the coefficients a, b_1, b_2 ?

[**2**] Consider the implicit Euler's method (or the backwards Euler's method)

$$y_{i+1} = y_i + hf(x_{i+1}, y_{i+1}).$$

Derive the region of absolute stability for the method. Given an ODE for which $\frac{\partial f}{\partial y} > 0$, does backwards Euler always give the qualitatively correct solution ? Explain.

[**3**] Find the interval of absolute stability for the trapezoidal method.

[**4**] The problem

$$\frac{\partial y}{\partial x} = \sqrt{y},\ y(0) = 0$$

has the nontrivial solution $y(x) = (x/2)^2$ yet an application of Euler's method to this problem yields $y_i = 0$ for all i and any h. Explain why this result doesn't violate the convergence theorem for Euler's method.

[**5**] Check wether or not the linear multistep method

$$y_i - y_{i-4} = \frac{h}{3}\Big[8f_{i-1} - 4f_{i-2} + 8f_{i-3}\Big]$$

is convergent, where $f_i = f(x_i, y_i)$.