MATH 269A

HW #2 (due Monday, October 15)

1. Show that the one-step method given by

$$\begin{aligned}
&\Phi(x,y,h) := \frac{1}{6}[k_1 + 4k_2 + k_3],\\
&k_1 := f(x,y),\\
&k_2 := f(x+\frac{h}{2}, y+\frac{h}{2}k_1),\\
&k_3 := f(x+h, y+h(-k_1+2k_2))
\end{aligned}$$

is consistent of third order.

2. Consider the one-step method given by

$$\Phi(x,y,h) := f(x,y) + \frac{h}{2}g(x+\frac{1}{3}h, y+\frac{1}{3}hf(x,y)),$$

where

$$g(x,y) := \frac{\partial}{\partial x}f(x,y) + (\frac{\partial}{\partial y}f(x,y))f(x,y).$$

Show that it is a consistent method of order 3.

3. Consider the (IVP)

$$y' = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1.$$

(a) Implement the classical fourth-order Runge-Kutta method and use it to numerically solve the (IVP).

(b) Give the solution and the error at $x = 2$ using stepsizes $h = 0.1$, $h = 0.05$, $h = 0.025$.

(c) How are these results compared with those obtained in HW#1 using Euler's method ?