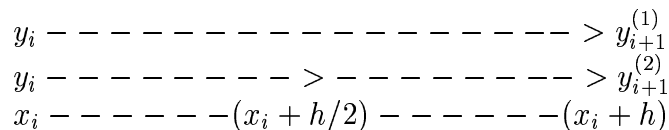


**Math 269A, Fall 2002.**

**HW # 6. Due Friday, November 22.**

[1] Consider using Euler's method for solving  $y' = f(y)$  and advancing the solution from  $y_i$  to  $y_{i+1}$  twice - once with a stepsize of  $h$  and another with a stepsize of  $h/2$ . The scheme below illustrates this:  $y_{i+1}^{(1)}$  is obtained with a step of  $h$ , while  $y_{i+1}^{(2)}$  is obtained with two steps of size  $h/2$ .



(a) Let  $y_{i+1}^{(*)} = \alpha y_{i+1}^{(1)} + \beta y_{i+1}^{(2)}$ . Without using the asymptotic error expansion, how should the values of  $\alpha$  and  $\beta$  be chosen so that  $y_{i+1}^{(*)}$  is a more accurate solution to the differential equation than either  $y_{i+1}^{(1)}$  or  $y_{i+1}^{(2)}$ ? Justify your answer (obtain the same result like in class, but by a different method, without using the asymptotic error expansion; use local truncation error and Taylor's expansion).

(b) For your choice of  $\alpha$  and  $\beta$  what is the order of the local truncation error associated with the scheme that advances the solution using  $y_{i+1}^{(*)} = \alpha y_{i+1}^{(1)} + \beta y_{i+1}^{(2)}$ ?

[2] (a) Give the solution (e.g. explicit formulas for  $y_1(x)$  and  $y_2(x)$ ) to

$$\frac{\partial}{\partial x} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \end{pmatrix}.$$

Give an estimate of the stepsize required to obtain a qualitatively correct solution if one is using Euler's method.

[3] Consider the second order differential equation

$$y'' + 19y' - 20y = 0.$$

(a) Give an equivalent first order system for this equation.

(b) Give the stability stepsize restriction if backward Euler is used to compute solutions to the first order system.