Math 269A, Fall 2002.
HW # 6. Due Friday, November 22.

[1] Consider using Euler's method for solving $y' = f(y)$ and advancing the solution from y_i to y_{i+1} twice - once with a stepsize of h and another with a stepsize of $h/2$. The scheme below illustrates this: $y_{i+1}^{(1)}$ is obtained with a step of h, while $y_{i+1}^{(2)}$ is obtained with two steps of size $h/2$.

$$\begin{array}{l} y_i - - - - - - - - - - - - - - - - - - - > y_{i+1}^{(1)} \\ y_i - - - - - - - - - > - - - - - - - - - > y_{i+1}^{(2)} \\ x_i - - - - - - (x_i + h/2) - - - - - - (x_i + h) \end{array}$$

(a) Let $y_{i+1}^{(*)} = \alpha y_{i+1}^{(1)} + \beta y_{i+1}^{(2)}$. Without using the asymptotic error expansion, how should the values of α and β be chosen so that $y_{i+1}^{(*)}$ is a more accurate solution to the differential equation than either $y_{i+1}^{(1)}$ or $y_{i+1}^{(2)}$? Justify your answer (obtain the same result like in class, but by a different method, without using the asymptotic error expansion; use local truncation error and Taylor's expansion).

(b) For your choice of α and β what is the order of the local truncation error associated with the scheme that advances the solution using $y_{i+1}^{(*)} = \alpha y_{i+1}^{(1)} + \beta y_{i+1}^{(2)}$?

[2] (a) Give the solution (e.g. explicit formulas for $y_1(x)$ and $y_2(x)$) to

$$\frac{\partial}{\partial x}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \end{pmatrix}.$$

Give an estimate of the stepsize required to obtain a qualitatively correct solution if one is using Euler's method.

[3] Consider the second order differential equation

$$y'' + 19y' - 20y = 0.$$

(a) Give an equivalent first order system for this equation.

(b) Give the stability stepsize restriction if backward Euler is used to compute solutions to the first order system.