MATH 269A, Fall 2002. HW #4 (due Friday, October 25)

1.

(a) Determine the coefficients in

$$y_{i+1} = a_0 y_i + h\{b_1 f(x_{i+1}, y_{i+1}) + b_0 f(x_i, y_i)\}$$

if the formula is to be exact for polynomials of degree 2 (consider polynomials x^j, for $j = 0,\ 1,\ 2$).

(b) Using the coefficients found in (a), find the order of the method and a simple expression of the local truncation error (use Taylor's expansion and express the local truncation error function of a higher derivative of the exact solution).

(c) What equivalent definition for the order of the method can be formulated ?

2. Use the quadratic interpolant to $f(x, y(x))$ at $x_i,\ x_{i-1},\ x_{i-2}$ to obtain the numerical method

$$y_{i+1} = y_{i-3} + \frac{4h}{3}\{2f(x_i, y_i) - f(x_{i-1}, y_{i-1}) + 2f(x_{i-2}, y_{i-2})\}.$$

What is the order of the local truncation error of the method ?

3. For $q = 1$ determine the coefficients β_{qi} in Nyström's formula

$$y_{p+1} = y_{p-1} + h\{\beta_{10} f(x_p, y_p) + \beta_{11} f(x_{p-1}, y_{p-1})\}.$$

4. Implement the fourth order Adams predictor-corrector method (sample code provided):

$$y_{i+1}^{(0)} = y_i + \frac{h}{24}\{55f(x_i, y_i) - 59f(x_{i-1}, y_{i-1}) + 37f(x_{i-2}, y_{i-2}) - 9f(x_{i-3}, y_{i-3})\}$$

$$y_{i+1} = y_i + \frac{h}{24}\{9f(x_{i+1}, y_{i+1}^{(0)}) + 19f(x_i, y_i) - 5f(x_{i-1}, y_{i-1}) + f(x_{i-2}, y_{i-2})\}$$

Verify the correctness of your implementation by applying it to the problem

$$y'(x) = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1.$$

(a) Explain how you obtained your starting values.

(b) Give the results at $x = 2$ of your algorithm for stepsizes $h = 0.1,\ 0.05,\ 0.025$, and 0.0125, and compare them with the exact solution at $x = 2$.