MATH 269A, Fall 2002
HW #2 (due Friday, October 11)

1. Show that the one-step method given by

$$\Phi(x, y, h) := \frac{1}{6}[k_1 + 4k_2 + k_3],$$
$$k_1 := f(x, y),$$
$$k_2 := f(x + \frac{h}{2}, y + \frac{h}{2}k_1),$$
$$k_3 := f(x + h, y + h(-k_1 + 2k_2))$$

is of third order.

2. Consider the one-step method given by

$$\Phi(x, y, h) := f(x, y) + \frac{h}{2}g(x + \frac{1}{3}h, y + \frac{1}{3}hf(x, y)),$$

where
$$g(x, y) := \frac{\partial}{\partial x}f(x, y) + (\frac{\partial}{\partial y}f(x, y))f(x, y).$$

Show that it is a method of order 3.

3. Consider the (IVP)

$$y' = \frac{3x^2 + 4x + 2}{2(y - 1)}, \quad y(0) = -1.$$

(a) Implement the classical fourth-order Runge-Kutta method and use it to numerically solve the (IVP).

(b) Give the solution and the error at $x = 2$ using stepsizes $h = 0.1$, $h = 0.05$, $h = 0.025$.

(c) How are these results compared with those obtained in HW#1 using Euler's method ?