MATH 269A, Fall 2002

HW #1 (due Monday, October 7)

1. Consider the initial-value problem

$$y' = x - x^2, \quad y(0) = 0.$$

Suppose we use Euler's method with stepsize h to compute approximate values y_i for $y(x_i)$, $x_i = ih$.

(a) Find an explicit formula for y_i and for $e(x_i, h) = y_i - y(x_i)$.

(b) Show that $e(x, h)$, for x fixed, goes to zero as $h \to 0$.

2. Given $\theta \in [0, 1]$, find the order of the method

$$y_{i+1} = y_i + hf(x_i + (1-\theta)h, \theta y_i + (1-\theta)y_{i+1}).$$

3. Provided that f is analytic, it is possible to obtain from $y' = f(x, y)$ an expression for the second derivative of y, namely $y'' = g(x, y)$, where

$$g(x, y) = \frac{\partial f(x, y)}{\partial x} + \frac{\partial f(x, y)}{\partial y} f(x, y).$$

Find the order of the method

$$y_{i+1} = y_i + hf(x_i, y_i) + \frac{1}{2}h^2 g(x_i, y_i).$$

4. Consider the (IVP)

$$y' = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1.$$

(a) By the method of separation of variables, find the exact solution.

(b) Implement Euler's method and use it to numerically solve the (IVP).

(c) Give the solution and the error at $x = 2$ using stepsizes $h = 0.1$, $h = 0.05$, $h = 0.025$.