5. Consider the linear programming problem below (in standard form). (a) Show that $x = (2, 1, 3, 0, 2)^T$ is a feasible solution to the problem. (b) Show that $d = (1, 3, 4, 1, 5)^T$ is a direction of unboundedness for the problem. (c) Use parts (a) and (b) to obtain a feasible solution $x' = x + \gamma d$ to the problem such that the value of the objective function at x' equals -100. (d) Generalize part (c) to show that the problem has no optimal solutions.

Minimize $z = 3x_1 - 2x_3$
subject to

$$\begin{aligned} 2x_1 - x_2 \quad + x_4 \quad &= 3 \\ 3x_1 \quad - 2x_3 \quad + x_5 &= 2 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

a) $2 \cdot 2 - 1 + 0 = 3$
$3 \cdot 2 - 2 \cdot 3 + 2 = 2$
and we can see that every entry from the vector $(2, 1, 3, 0, 2)^T$ is a ~~#~~ non-negative number, so $(2, 1, 3, 0, 2)^T$ is a feasible solution.

b) Let $(x_1, x_2, x_3, x_4, x_5)^T$ be a feasible solution of the above problem.
Let $\gamma > 0$.
We will show that $(x_1, x_2, x_3, x_4, x_5)^T + \gamma \cdot (1, 3, 4, 1, 5)^T$ is a feasible solution.

$(x_1, x_2, x_3, x_4, x_5)^T + \gamma \cdot (1, 3, 4, 1, 5)^T = (x_1 + \gamma, x_2 + 3\gamma, x_3 + 4\gamma, x_4 + \gamma, x_5 + 5\gamma)$

$2(x_1 + \gamma) - (x_2 + 3\gamma) + (x_4 + \gamma) = 2x_1 - x_2 + x_4 = 3$

BESIDE THIS SOLUTION WE CAN USE THE OTHER METHOD WHICH INVOLVE THE CHARACTERIZATION OF A DIRECTION OF UNBOUNDEDNESS.