Math 164, Lecture 2, Vese

Homework #7, due on <u>WEDNESDAY</u>, March 1st

Please review sections 6.1, 6.2 (except proof of Thm. 6.2), and 6.2.1.
Please read subsection 6.2.2.

Problems:

[**1**] Consider the linear program

$$\text{maximize } z = -x_1 - x_2, \text{ subject to } \begin{cases} -x_1 + x_2 \geq 1 \\ 2x_1 - x_2 \leq 2 \\ x_1, x_2 \geq 0. \end{cases}$$

Find the dual to the problem. Solve the primal and the dual graphically, and verify that the results of the strong duality theorem hold.

[**2**] Prove that if both the primal and the dual linear problems have feasible solutions, then both have optimal solutions, and the optimal objective values of the two problems are equal.

[**3**] Prove the following Corollary from Section 6.2, and use the Weak Duality Theorem:
Corollary 6.2: *If x is a feasible solution to the primal, and y is a feasible solution to the dual, and $c^T x = b^T y$, then x and y are optimal for their respective problems.*

[**4**] Consider the linear program

$$\begin{array}{rc} \text{minimize} & z = 2x_1 + 9x_2 + 3x_3 \\ \text{subject to} & -2x_1 + 2x_2 + x_3 \geq 1 \\ & x_1 + 4x_2 - x_3 \geq 1 \\ & x_1,\ x_2,\ x_3 \geq 0. \end{array}$$

(a) Find the dual to this problem and solve the dual problem graphically.
(b) Use complementarity slackness to obtain the solution to the primal.

[**5**] Consider the primal linear programming problem
Minimize $z = c^T x$
subject to $Ax \leq b$,
$x \geq 0$.

Assume that this problem and its dual are both feasible. Let x_* be an optimal solution vector to the primal, let z_* be its associated objective value, and let y_* be an optimal solution vector to the dual problem. Show that

$$z_* = y_*^T A x_*.$$