

Math 164, Vese

Homework #6, due on Friday, February 17, 2006

Please review sections 5.2 and 6.1

Problems:

- [1] Consider the linear program: Minimize $z = x_1 - x_2$ subject to

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ x_1 - 2x_2 &\leq 2 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Derive an expression for the set of optimal solutions to this problem, and show that this set is unbounded.

- [2] Consider the linear program of problem [4] from the previous homework, recalled here:

$$\text{minimize } z = -5x_1 - 7x_2, \text{ subject to } \begin{cases} -3x_1 + 2x_2 \leq 30 \\ -2x_1 + x_2 \leq 12 \\ x_1, x_2 \geq 0. \end{cases}$$

Show by the method of your choice that this problem has no finite optimal solution.

- [3] Find the dual of

$$\begin{aligned} \text{minimize } z &= 3x_1 - 5x_2 - 7x_3, \\ \text{subject to} \\ x_1 + 5x_2 - 8x_3 &\geq 5 \\ 4x_1 - 2x_2 + 7x_3 &\geq 7 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

- [4] Find the dual of

$$\begin{aligned} \text{maximize } z &= 6x_1 - 3x_2 - 2x_3 + 5x_4, \\ \text{subject to} \\ 4x_1 + 3x_2 - 8x_3 + 7x_4 &= 11 \\ 3x_1 + 2x_2 + 7x_3 + 6x_4 &\geq 23 \\ 7x_1 + 4x_2 + 3x_3 + 2x_4 &\leq 12 \\ x_1, x_2 &\geq 0, x_3 \leq 0, x_4 \text{ free} \end{aligned}$$

Verify that the dual of the dual is the primal.

- [5] Find the dual to the problem

$$\begin{aligned} \text{minimize } z &= c^T x, \\ \text{subject to } b_1 &\leq Ax \leq b_2, \\ x &\geq 0. \end{aligned}$$