Math 164, Homework #5, due on Friday, February 10, 2006

Remarks:

• REMINDER: midterm on Friday, February 10, 12-12.50pm (MS 5137). This will be a closed note and closed book written examination.

• Sections covered for the midterm: 1.2-1.5, 2.2, 2.3 (except 2.3.1), 3.1, 4.1, 4.2, 4.3, 4.4, 5.2 (except 5.2.1 and 5.2.2).

• Office hours with the instructor Vese before the midterm: Monday Feb. 6, time 2-4pm, and Wednesday Feb. 8, time 2-4pm (MS 7620-D).

• There are no office hours with the instructor Vese on Thursday Feb. 9 or on Friday Feb. 10. For questions on Thursday or Friday, please contact the teaching assistant Salazar (MS 2951).

• Sample practice problems with solutions for the midterm posted on the class webpage.

[**1**] Convert the following linear program to standard form:

$$\text{minimize } z = x_1 - 5x_2 - 7x_3, \text{ subject to } \begin{cases} 5x_1 - 2x_2 + 6x_3 \geq 5 \\ 3x_1 + 4x_2 - 9x_3 = 3 \\ 7x_1 + 3x_2 + 5x_3 \leq 9 \\ x_1 \geq -2, \ x_2, \ x_3 \text{ free.} \end{cases}$$

[**2**] Consider a linear program with the constraints in standard form

$$Ax = b \text{ and } x \geq \vec{0}.$$

(a) Prove that, if d is a direction of unboundedness for these constraints, then $-d$ cannot be a direction of unboundedness.

(b) Let $\{d_1, ..., d_k\}$ be directions of unboundedness for these constraints. Prove that a nonzero vector $d = \sum_{i=1}^{k} \alpha_i d_i$, with $\alpha_i \geq 0$ is also a direction of unboundedness.

[**3**] Suppose that a linear program in standard form, with bounded feasible region, has l optimal extreme points $\{v_1, v_2, ..., v_l\}$. Prove that a point is optimal for the linear program if, and only if, it can be expressed as a convex combination of $\{v_1, v_2, ..., v_l\}$.

[**4**] Consider the linear program: minimize $z = -5x_1 - 7x_2$, subject to $\begin{cases} -3x_1 + 2x_2 \leq 30 \\ -2x_1 + x_2 \leq 12 \\ x_1, x_2 \geq 0. \end{cases}$

(a) Draw a graph of the feasible region and determine two linearly-independent directions of unboundedness.

(b) Represent the point $x = (6, 12)^T$ as a convex combination of extreme points, plus if applicable, a direction of unboundedness.

(c) Convert the linear program to standard form and determine two linearly-independent directions of unboundedness for this version of the problem. Verify that the directions of unboundedness satisfy $Ad = \vec{0}$ and $d \geq \vec{0}$.

[**5**] Solve the following linear program using the simplex method (graph the feasible region, and outline the progress of the solution).

$$\text{maximize } z = 7x_1 + 8x_2, \text{ subject to } \begin{cases} 4x_1 + x_2 \leq 100 \\ x_1 + x_2 \leq 80 \\ x_1 \leq 40 \\ x_1, x_2 \geq 0. \end{cases}$$