Math 164, Lecture 2, Vese
Homework #3, due on Friday, January 27, 2005

Remarks:

• Please note that exceptionally, there is no office hour with the instructor on Wednesday, January 25.

• Review Sections 2.3 (except 2.3.1) and 3.1 from the textbook.

[**1**] Prove that a function f is concave if and only if $-f$ is convex.

[**2**] Let f be a convex function on the convex set S of R^n. Let k be a nonzero scalar, and define $g(x) = kf(x)$. Prove that if $k > 0$ then g is a convex function on S, and if $k < 0$ then g is a concave function of S.

[**3**] Consider a feasible region S defined by a set of linear constraints

$$S = \{x : \ Ax \leq b\},$$

where A is an $m \times n$ matrix and b is a column vector.

(a) Prove that S is convex.

(b) Derive the conditions that must be satisfied by a feasible direction p at a point $x \in S$.

[**4**] Consider the problem

$$\text{minimize } f(x)$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 = 6, \ x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0.$$

(a) Find the sets of all feasible directions at points $x_a = (0, 0, 2)^T$, $x_b = (3, 0, 1)^T$, and $x_c = (1, 1, 1)^T$.

(b) Using (a), verify that $p = (3, 0, -1)^T$ is a feasible direction for $x_c = (1, 1, 1)^T$; then find an upper bound on the step length α so that $x_c + \alpha p$ is a feasible point, with $p = (3, 0, -1)^T$.