Minimize $z = -x_1 + 2x_2 - 3x_4$
subject to

$$\begin{array}{rcl}
x_1 \qquad + 2x_3 - x_4 & \geq & 1 \\
x_2 - x_3 \qquad & \geq & 0 \\
2x_1 - 2x_2 \qquad - x_4 & \geq & -3 \\
x_3 + 2x_4 & \geq & 3 \\
-3x_1 \qquad + x_4 & \geq & -1 \\
x_1, x_2, x_3, x_4 & \geq & 0
\end{array}$$

4. (a) Define what is mean by "the function f defined on the convex set S is a convex function". (b) Let f and g be convex functions on a convex set S. Define a function h by

$$h(x) = rf(x) + sg(x)$$

for all x in S, where r and s are positive real numbers. Prove that h is also a convex function on S, making it clear in your proof how you are making use of the hypothesis that r and s are positive.

5. Consider the linear programming problem below (in standard form). (a) Show that $x = (2, 1, 3, 0, 2)^T$ is a feasible solution to the problem. (b) Show that $d = (1, 3, 4, 1, 5)^T$ is a direction of unboundedness for the problem. (c) Use parts (a) and (b) to obtain a feasible solution $x' = x + \gamma d$ to the problem such that the value of the objective function at x' equals -100. (d) Generalize part (c) to show that the problem has no optimal solutions.

Minimize $z = 3x_1 - 2x_3$
subject to

$$\begin{array}{rcl}
2x_1 - x_2 \qquad + x_4 \qquad & = & 3 \\
3x_1 \qquad - 2x_3 \qquad + x_5 & = & 2 \\
x_1, x_2, x_3, x_4, x_5 & \geq & 0
\end{array}$$