Math 164, Lecture 2, Vese
Homework #7, due on Wednesday, November 16, 2005

Important remarks:

• Reminder: midterm exam on Wednesday, November 9, 1-1.50pm (in the usual lecture room, MS 6229). This will be a closed note and closed book written examination. You will write your midterm solutions only on the paper provided.

• Sections covered for the midterm: 1.2-1.5, 2.2-2.3, 3.1, 4.1-4.4, 5.2, and 6.1.

• Additional office hours with the instructor before the midterm:

- Monday Nov. 7 (time 2-3.45pm), in MS 7620-D.

- Please note: there are no office hours with the instructor Vese on Tuesday Nov. 8 or on Wednesday Nov. 9.

- Tuesday Nov. 8, the office hours will be only with the T.A. Tristan Roy.

• Sample midterm questions and solutions are posted on the class webpage.

• Handout on the simplex method is posted on the class webpage.

• Hw #6 is due as usual, this week Wednesday November 9 at the end of the midterm.

• Review sections 6.1, 6.2, and 6.2.1 before starting the homework.

Problems:

[**1**] Consider the linear program

$$\text{maximize } z = -x_1 - x_2, \text{ subject to } \begin{cases} -x_1 + x_2 \geq 1 \\ 2x_1 - x_2 \leq 2 \\ x_1, x_2 \geq 0. \end{cases}$$

Find the dual to the problem. Solve the primal and the dual graphically, and verify that the results of the strong duality theorem hold.

[**2**] Prove that if both the primal and the dual linear problems have feasible solutions, then both have optimal solutions, and the optimal objective values of the two problems are equal.

[**3**] Prove the following Corollaries from Section 6.2, and use the Weak Duality Theorem:

(a) **Corollary 6.1:** If the primal is unbounded, then the dual is infeasible. If the dual is unbounded, then the primal is infeasible.

(b) **Corollary 6.2:** If x is a feasible solution to the primal, and y is a feasible solution to the dual, and $c^T x = b^T y$, then x and y are optimal for their respective problems.

[**4**] Consider the linear program

$$\begin{array}{rc} \text{minimize} & z = 2x_1 + 9x_2 + 3x_3 \\ \text{subject to} & -2x_1 + 2x_2 + x_3 \geq 1 \\ & x_1 + 4x_2 - x_3 \geq 1 \\ & x_1,\ x_2,\ x_3 \geq 0. \end{array}$$

(a) Find the dual to this problem and solve it graphically.

(b) Use complementarity slackness to obtain the solution to the primal.