

**Math 164, Lecture 2, Vese**

**Homework #5, due on Wednesday, November 2nd, 2005**

Remarks:

- Please review Sections 4.4 and 5.2 from the textbook.
- Reminder: midterm exam on Wednesday, November 9, 1-2pm (MS 6229). This will be a closed note and closed book written examination.
- Sections covered for the midterm: 1.2-1.5, 2.2-2.3, 3.1, 4.1-4.4, 5.2, and 6.1.
- Office hours with the instructor before the midterm: Monday Nov. 7, time 2-4pm, in MS 7620-D. (please note: on Tuesday Nov. 8, the office hours will be only with the T.A. Tristan Roy).

[1] Consider a linear program with the constraints in standard form

$$Ax = b \text{ and } x \geq \vec{0}.$$

- (a) Recall the definition of a direction of unboundedness.
- (b) Prove that if  $d \neq \vec{0}$  satisfies  $Ad = \vec{0}$  and  $d \geq \vec{0}$ , then  $d$  is a direction of unboundedness for these constraints.
- (c) Prove that, if  $d$  is a direction of unboundedness for these constraints, then  $-d$  cannot be a direction of unboundedness.
- (d) Let  $\{d_1, \dots, d_k\}$  be directions of unboundedness for these constraints. Prove that a nonzero vector  $d = \sum_{i=1}^k \alpha_i d_i$ , with  $\alpha_i \geq 0$  is also a direction of unboundedness.

[2] Suppose that a linear program in standard form, with bounded feasible region, has  $l$  optimal extreme points  $\{v_1, v_2, \dots, v_l\}$ . Prove that a point is optimal for the linear program if, and only if, it can be expressed as a convex combination of  $\{v_1, v_2, \dots, v_l\}$ .

[3] Convert the following linear program to standard form:

$$\text{minimize } z = x_1 - 5x_2 - 7x_3, \text{ subject to } \begin{cases} 5x_1 - 2x_2 + 6x_3 \geq 5 \\ 3x_1 + 4x_2 - 9x_3 = 3 \\ 7x_1 + 3x_2 + 5x_3 \leq 9 \\ x_1 \geq -2, x_2, x_3 \text{ free.} \end{cases}$$

[4] Solve the following linear program using the simplex method (graph the feasible region, and outline the progress of the solution).

$$\text{maximize } z = 7x_1 + 8x_2, \text{ subject to } \begin{cases} 4x_1 + x_2 \leq 100 \\ x_1 + x_2 \leq 80 \\ x_1 \leq 40 \\ x_1, x_2 \geq 0. \end{cases}$$

[5] Consider the linear program

$$\text{minimize } z = -5x_1 - 7x_2, \text{ subject to } \begin{cases} -3x_1 + 2x_2 \leq 30 \\ -2x_1 + x_2 \leq 12 \\ x_1, x_2 \geq 0. \end{cases}$$

(a) Draw a graph of the feasible region and determine two linearly-independent directions of unboundedness.

(b) Convert the linear program to standard form and determine two linearly-independent directions of unboundedness for this version of the problem. Verify that the directions of unboundedness satisfy  $Ad = \vec{0}$  and  $d \geq \vec{0}$ .