Math 164, Lecture 2, Vese

Homework #4, due on Wednesday, October 26, 2005

Remarks:

• Each time, please solve as many problems as you can from the textbook

• Please review Sections 4.2 and 4.3 from the textbook.

[**1**] Consider the problem

$$\text{minimize } f(x)$$
$$\text{subject to } x_1 + 2x_2 + 3x_3 = 6,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,$$

and the point $x_c = (1, 1, 1)^T$.

(a) Show that x_c is a feasible point.

(b) Verify that $p = (3, 0, -1)^T$ is a feasible direction for x_c (see also your previous homework).

(c) Find an upper bound on the step length α so that $x_c + \alpha p$ is a feasible point, with $p = (3, 0, -1)^T$.

[**2**] Consider the following linear program (compare with Problem [3] from Hw#3):

minimize

$$z = 3x_1 + x_2,$$

subject to

$$\begin{aligned} -x_1 + x_2 &\geq -1, \\ 3x_1 + 2x_2 &\leq 12, \\ 2x_1 + 3x_2 &\leq 3, \\ x_1, x_2 &\geq 0. \end{aligned}$$

(a) Solve the problem graphically.

(b) Write the problem in standard form.

[**3**] Consider the system of linear constraints

$$\begin{array}{ccccc} 2x_1 & + & x_2 & \leq & 100, \\ x_1 & + & x_2 & \leq & 80, \\ x_1 & & & \leq & 40, \\ x_1 & , & x_2 & \geq & 0. \end{array}$$

(a) Write this system in standard form, and determine all the basic solutions (feasible and infeasible).

(b) Determine the extreme points of the feasible region (corresponding to both the standard form, as well as the original version).