

Math 164, Lecture 2, Vese
Homework #3, due on Wednesday, October 19, 2005

Remarks:

- each time, solve as many problems as you can from the textbook
- review Sections 3.1 and 4.1 from the textbook.
- T. A. Tristan Roy new office hours: Tuesdays 2-3pm (right after D.S.)

[1] Consider a feasible region S defined by a set of linear constraints

$$S = \{x : Ax \leq b\},$$

where A is an $m \times n$ matrix and b is a column vector.

- (a) Prove that S is convex.
- (b) Derive the conditions that must be satisfied by a feasible direction p at a point $x \in S$.

[2] Consider the problem

$$\text{minimize } f(x)$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 = 6, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Find the sets of all feasible directions at points $x_a = (0, 0, 2)^T$, $x_b = (3, 0, 1)^T$, and $x_c = (1, 1, 1)^T$.

[3] Solve the following linear program graphically:

minimize

$$z = 3x_1 + x_2,$$

subject to

$$\begin{aligned}x_1 - x_2 &\leq 1, \\3x_1 + 2x_2 &\leq 12, \\2x_1 + 3x_2 &\leq 3, \\-2x_1 + 3x_2 &\geq 9, \\x_1, x_2 &\geq 0.\end{aligned}$$