5. Consider the linear programming problem below (in standard form). (a) Show that $x = (2, 1, 3, 0, 2)^T$ is a feasible solution to the problem. (b) Show that $d = (1, 3, 4, 1, 5)^T$ is a direction of unboundedness for the problem. (c) Use parts (a) and (b) to obtain a feasible solution $x' = x + \gamma d$ to the problem such that the value of the objective function at $x'$ equals $-100$. (d) Generalize part (c) to show that the problem has no optimal solutions.

Minimize $z = 3x_1 - 2x_3$
subject to
\[
egin{align*}
2x_1 - x_2 + x_4 &= 3 \\
3x_1 - 2x_3 + x_5 &= 2 \\
x_1, x_2, x_3, x_4, x_5 &\geq 0
\end{align*}
\]

a) $2 \cdot 2 - 1 + 0 = 3$
\[3 \cdot 2 - 2 \cdot 3 + 2 = 2\]
and we can see that every entry from the vector $(2, 1, 3, 0, 2)^T$ is a non-negative number, so $(2, 1, 3, 0, 2)^T$ is a feasible solution.

b) Let $(x_1, x_2, x_3, x_4, x_5)^T$ be a feasible solution of the above problem. Let $\gamma > 0$.

We write $y = (1, 3, 4, 1, 5)^T$, a feasible solution.

\[
(x_1, x_2, x_3, x_4, x_5)^T + \gamma y = (x_1 + \gamma, x_2 + 3\gamma, x_3 + 4\gamma, x_4 + \gamma, x_5 + 5\gamma)
\]

\[
2(x_1 + \gamma) - (x_2 + 3\gamma) + (x_4 + \gamma) = 2x_1 - x_2 + x_4 = 3
\]

BESIDE THIS SOLUTION WE CAN USE THE OTHER METHOD WHICH INVOLVE THE CHARACTERIZATION OF A DIRECTION OF UNBOUNDEDNESS.