3. Consider the linear programming problem below:
Minimize $z = -x_1 + 2x_2 - 3x_4$
subject to

$$\begin{array}{rcrcrcrcr}
x_1 & & & + & 2x_3 & - & x_4 & \geq & 1 \\
 & & x_2 & - & x_3 & & & \geq & 0 \\
2x_1 & - & 2x_2 & & & - & x_4 & \geq & -3 \\
 & & & & x_3 & + & 2x_4 & \geq & 3 \\
-3x_1 & & & & & + & x_4 & \geq & -1 \\
x_1, & & x_2, & & x_3, & & x_4 & \geq & 0.
\end{array}$$

(a) Show that $x = (1, 1, 1, 2)^T$ is a feasible solution to the problem.

(b) Label each of the constraints of the problem as active or inactive for the feasible solution $x = (1, 1, 1, 2)^T$.

(c) Determine all values of a such that the vector $p = (-1, 1, 1, a)^T$ is a feasible direction at the feasible solution $x = (1, 1, 1, 2)^T$.

Solution:
(a) and (b):
For $x = (1, 1, 1, 2)^T$, we have:
$1 + 2 - 2 = 1$ (the first constraint is active)
$1 - 1 = 0$ (the second constraint is active)
$2 - 2 - 2 = -2 > -3$ (the third constraint is inactive)
$1 + 4 = 5 > 3$ (the fourth constraint is inactive)
$-3 + 2 = -1 = -1$ (the fifth constraint is active)
and $x_i > 0$, so $(1, 1, 1, 2)^T$ is a feasible point or feasible solution.