2. **Consider the linear programming problem below (in standard form). (a) Write the basic feasible solution corresponding to the basis $\{x_2, x_3\}$. (b) Determine whether the solution of part (a) is optimal.**

Minimize $z = -3x_1 + 5x_2 + x_3$
subject to

$$\begin{aligned} 4x_1 + 3x_2 \quad + x_4 \quad &= 2 \\ -2x_2 + x_3 \quad + x_5 &= 1 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

Set the nonbasic variables x_1, x_4, x_5 to be zero.

$x_1 = x_4 = x_5 = 0$

Solve the system for the basic variables

$$\begin{cases} 3x_2 = 2 & \Rightarrow x_2 = \frac{2}{3} \\ -2x_2 + x_3 = 1 & \Rightarrow x_3 = 1 + \frac{4}{3} = \frac{7}{3} \end{cases}$$

The basic feasible $(0, \frac{2}{3}, \frac{7}{3}, 0, 0)$

$x_B = \{x_2, x_3\}$, $x_N = \{x_1, x_4, x_5\}$

$3x_2 = 2 - 4x_1 - x_4 \implies x_2 = \frac{2}{3} - \frac{4}{3}x_1 - \frac{1}{3}x_1$

$x_3 = 1 + 2x_2 - x_5$

$= 1 + 2\left(\frac{2}{3} - \frac{4}{3}x_1 - \frac{1}{3}x_4\right) - x_5$

$$\begin{cases} x_3 = \frac{7}{3} - \frac{8}{3}x_1 - \frac{2}{3}x_4 - x_5 \\ x_2 = \frac{2}{3} - \frac{4}{3}x_1 - \frac{1}{3}x_4 \end{cases}$$

$z = -3x_1 - 5\left(\frac{2}{3} - \frac{11}{3}x_1 - \frac{1}{3}x_4\right) + \left(\frac{7}{3} - \frac{8}{3}x_1 - \frac{1}{3}x_4 - x_5\right)$

$= \left(-3 + \frac{55}{3} - \frac{8}{3}\right)x_1 + \left(\frac{5}{3} - \frac{2}{3}\right)x_4 - x_5 - \frac{10}{3}$

Not optimal.