1. Convert the following linear programming problem to standard form and then write out the coefficient matrix A.

Maximize $3x_1 - x_2 + 4$
subject to

$$\begin{aligned} x_1 - x_2 &\geq 1 \\ -2x_1 + 3x_2 &= 3 \\ x_1 \leq 2, x_2 & \ free \end{aligned}$$

STEP 1° MAKE ALL THE VARIABLES NON-NEGATIVE

$x_1 \leq 2 \Rightarrow 2 - x_1 \geq 0$; $x_3 = 2 - x_1 \Rightarrow x_1 = 2 - x_3$

x_2 free $\Rightarrow x_2 = x_4 - x_5$, $x_4 \geq 0$, $x_5 \geq 0$

$$\begin{aligned} \text{Max } & 3(2-x_3) - (x_4 - x_5) + 4 \\ \text{s.t. } & (2-x_3) - (x_4 - x_5) \geq 1 \\ & -2(2-x_3) + 3(x_4 - x_5) = 3 \\ & x_3, x_4, x_5 \geq 0 \end{aligned}$$

which is equivalent with

$$\begin{aligned} \text{Max } & -3x_3 - x_4 + x_5 + 10 \\ \text{s.t. } & \begin{cases} -x_3 - x_4 + x_5 \geq -1 \\ 2x_3 + 3x_4 - 3x_5 = 7 \end{cases} \\ & x_3, x_4, x_5 \geq 0 \end{aligned}$$

STEP 2° Minimum Problem:

Min $3x_3 + x_4 - x_5 - 10$
s.t. the same constraints

STEP 3° Remove the constant from the objective function

Min $3x_3 + x_4 - x_5$
s.t. the same constraints

STEP 4° Make all the constraints values ("b") non-negative