1. Convert the following linear programming problem to standard form and then write out the coefficient matrix A.

Maximize $3x_1 - x_2 + 4$
subject to

$$\begin{aligned} x_1 - x_2 &\geq 1 \\ -2x_1 + 3x_2 &= 3 \\ x_1 \leq 2, x_2 &\ free \end{aligned}$$

2. Consider the linear programming problem below (in standard form). (a) Write the basic feasible solution corresponding to the basis $\{x_2, x_3\}$. (b) Determine whether the solution of part (a) is optimal.

Minimize $z = -3x_1 - 5x_2 + x_3$
subject to

$$\begin{aligned} 4x_1 + 3x_2 + x_4 &= 2 \\ -2x_2 + x_3 + x_5 &= 1 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

3. Consider the primal linear programming problem in canonical form below. (a) Show that $x = (1, 1, 1, 2)^T$ is a feasible solution to the problem. (b) Label each of the constraints of the problem as active or inactive for the feasible solution $x = (1, 1, 1, 2)^T$. (c) Determine all values of a such that the vector $p = (-1, 1, 1, a)^T$ is a feasible direction at the feasible solution $x = (1, 1, 1, 2)$. (d) Write the objective function of the dual linear programming problem, but no other part of that problem. (Note that part (d) has nothing to do with any other part of this question.)