Math 164 - Handout Used for 5.2.1 - General Formulae for the Simplex Method

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Consider the linear programming problem in standard form

$$\left.\begin{array}{lrcl} \text{minimize} & z & = & c^T x \\ \text{subject to} & Ax & = & b \\ & x & \geq & \mathcal{O} \end{array}\right\} \quad (*)$$

with $b \geq \mathcal{O}$.

1. Reordering

   Write[1]

$$x = \begin{bmatrix} x_B \\ x_N \end{bmatrix} \qquad \text{and} \qquad A = \begin{bmatrix} B & N \end{bmatrix}$$

   then

$$\begin{aligned} z &= c^T x \\ &= \begin{bmatrix} c_B^T & c_N^T \end{bmatrix} \begin{bmatrix} x_B \\ x_N \end{bmatrix} \\ &= c_B^T x_B + c_N^T x_N \end{aligned} \qquad (1)$$

   We then can rewrite problem $(*)$ as follows

$$\begin{array}{rll} \text{minimize} & z = c_B^T x_B + c_N^T x_N & (2) \\ \text{subject to} & B x_B + N x_N = b & (3) \\ & x \geq \mathcal{O} & \end{array}$$

   B is basis matrix, hence non-singular, i.e. B^{-1} exists.

[1] see section 4.3 for notation

2. Express Basic Variables and Objective in terms of the Non-Basic Variables

Solving (3) for x_B yields

$$x_B = B^{-1}b - B^{-1}Nx_N. \tag{4}$$

Using (4) in (1) yields

$$\begin{aligned} z &= c_B^T x_B + c_N^T x_N \\ &= c_B^T(B^{-1}b - B^{-1}x_N) + c_N^T x_N \quad \text{by (4)} \\ &= \underbrace{c_B^T B^{-1}}_{\stackrel{\text{def}}{=} y^T} b + (c_N^T - \underbrace{c_B^T B^{-1}}_{\stackrel{\text{def}}{=} y^T} N)x_N \end{aligned}$$