9. Consider the following linear programming problem.

Minimize $z = -x_1 + 3x_2 + x_4 + x_5$
subject to

$$\begin{aligned} x_1 + 2x_2 + \tfrac{3}{4}x_3 + x_4 + x_5 &= 1 \\ -2x_1 + 3x_2 + x_3 + x_4 &= 1 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

At a certain step in solving this problem by the simplex method, the basis is $\{x_2, x_4\}$. Determine the next basis according to the rules of the simplex method, showing your work in detail.

$x_4 = 1 + 2x_1 - 3x_2 - x_3$

$x_1 + 2x_2 + \frac{3}{4}x_3 + (1 + 2x_1 - 3x_2 - x_3) + x_5 = 1$

$x_1 - x_2 - \frac{1}{4}x_3 + x_5 = 0 \qquad x_2 = x_1 - \frac{1}{4}x_3 + x_5$

$x_4 = 1 + 2x_1 - 3(x_1 - \frac{1}{4}x_3 + x_5) - x_3 = 1 - x_1 - \frac{1}{4}x_3 - 3x_5$

$z = -x_1 + 3(x_1 - \frac{1}{4}x_3 + x_5) + (1 - x_1 - \frac{1}{4}x_3 - 3x_5) + x_5$

$= x_1 - x_3 + x_5$ so x_3 enters the basis

setting $x_1 = x_5 = 0$

$x_2 = -\frac{1}{4}x_3 \geq 0$ only if $x_3 = 0$

$x_4 = 1 - \frac{1}{4}x_3 \geq 0$ if $x_3 \leq 4$

so x_2 must leave the basis and the new basis is $\{x_3, x_4\}$