4. Show that $(1,1)^T$ is a local minimizer for the problem

Minimize $f(x) = f(x_1, x_2) = x_1^2 + 2x_2^2 - x_1 x_2$
subject to

active	$x_1 + x_2 \geq 2$	$g_1(x) = x_1 + x_2 - 2$
active	$x_2 \geq x_1$	$g_2(x) = -x_1 + x_2$
inactive	$x_2 \leq 2$	$g_3(x) = -x_2 + 2$

by demonstrating that $(1,1)^T$ satisfies the sufficient conditions presented in the course.

$\lambda_3 = 0$ $\quad A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$ $\quad \nabla f(x) = \begin{bmatrix} 2x_1 - x_2 \\ 4x_2 - x_1 \end{bmatrix}$

$$\nabla f(1,1) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda_1 - \lambda_2 \\ \lambda_1 + \lambda_2 \end{bmatrix}$$

$\lambda_1 = 2$; $\lambda_2 = 1$, $\lambda_1, \lambda_2 > 0$

$$\nabla^2 f(x) = \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -1 \\ 0 & 7/2 \end{bmatrix} \text{ pos. def.}$$

Note $\hat{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ is non-singular so there is no Z.