3. Using Newton's method to find the stationary points of

$$f(x) = f(x_1, x_2) = x_1^4 - 2x_1 - x_1^2 x_2 - x_2^3$$

we start with $\bar{x}_0 = (-1/2, 1/2)^T$. Calculate $\bar{x}_1$ in fraction form.

$$\nabla f(x) = \begin{bmatrix} 4x_1^3 - 2 - 2x_1 x_2 \\ -x_1^2 - 3x_2^2 \end{bmatrix} = \begin{bmatrix} 4(-\frac{1}{2})^3 - 2 - 2(-\frac{1}{2})(\frac{1}{2}) \\ -(-\frac{1}{2})^2 - 3(\frac{1}{2})^2 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} - 2 + \frac{1}{2} \\ -\frac{1}{4} - \frac{3}{4} \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 12x_1^2 - 2x_2 & -2x_1 \\ -2x_1 & -6x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 12(-\frac{1}{2})^2 - 2(\frac{1}{2}) & -2(-\frac{1}{2}) \\ -2(-\frac{1}{2}) & -6(\frac{1}{2}) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$$

$$(\nabla^2 f(x))^{-1} = \begin{bmatrix} 3/7 & 1/7 \\ 1/7 & -2/7 \end{bmatrix}$$

$$\bar{x}_1 = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} - \begin{bmatrix} 3/7 & 1/7 \\ 1/7 & -2/7 \end{bmatrix} \begin{bmatrix} -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$