2. Show that $(-2/11, 6/11, -2/11)^T$ is a local minimizer for the problem

Minimize $f(x) = f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$
subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ -x_1 + 3x_2 - x_3 &= 2 \end{aligned}$$

by demonstrating that $(-2/11, 6/11, -2/11)^T$ satisfies the sufficient conditions presented in the course.

feasible: $2\left(-\frac{2}{11}\right) + \frac{6}{11} - \frac{2}{11} = 0$

$-\left(-\frac{2}{11}\right) + 3\left(\frac{6}{11}\right) - \left(-\frac{2}{11}\right) = \frac{22}{11} = 2$

$\nabla f(x) = \begin{bmatrix} 2x_1 \\ 2x_2 \\ 2x_3 \end{bmatrix}$ $\quad Ap = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 3 & -1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2p_1 + p_2 + p_3 = 0 \quad -p_1 + 3p_2 - p_3 = 0 \quad p_3 = -p_1 + 3p_2$

$2p_1 + p_2 + (-p_1 + 3p_2) = 0 \quad p_1 = -4p_2 \quad p_3 = 7p_2$

so $Z = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$ $\quad Z^T \nabla f(-2/11, 6/11, -2/11) = [-4 \ \ 1 \ \ 7] \begin{bmatrix} -4/11 \\ 12/11 \\ -4/11 \end{bmatrix} = 0$

$\nabla^2 f(x) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ positive definite so

$Z^T \nabla^2 f(x) Z$ is also.