by demonstrating that $(0, -1)^T$ satisfies the sufficient conditions presented in the course.

9. Consider the following linear programming problem.

Minimize $z = -x_1 + 3x_2 + x_4 + x_5$
subject to

$$\begin{aligned} x_1 + 2x_2 + \tfrac{3}{4}x_3 + x_4 + x_5 &= 1 \\ -2x_1 + 3x_2 + x_3 + x_4 \quad &= 1 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

At a certain step in solving this problem by the simplex method, the basis is $\{x_2, x_4\}$. Determine the next basis according to the rules of the simplex method, showing your work in detail.

10. Prove that if x_* is a vector such that $\nabla f(x_*) = \mathbf{0}$ and $\nabla^2 f(x_*)$ is positive definite, then x_* is a local minimizer for $f(x)$.