by demonstrating that \((0, -1)^T\) satisfies the sufficient conditions presented in the course.

9. Consider the following linear programming problem.

Minimize \(z = -x_1 + 3x_2 + x_4 + x_5\)

subject to

\[
\begin{align*}
    x_1 + 2x_2 + \frac{3}{4}x_3 + x_4 + x_5 & = 1 \\
    -2x_1 + 3x_2 + x_3 + x_4 & = 1 \\
    x_1, x_2, x_3, x_4, x_5 & \geq 0
\end{align*}
\]

At a certain step in solving this problem by the simplex method, the basis is \(\{x_2, x_4\}\). Determine the next basis according to the rules of the simplex method, showing your work in detail.

10. Prove that if \(x_*\) is a vector such that \(\nabla f(x_*) = 0\) and \(\nabla^2 f(x_*)\) is positive definite, then \(x_*\) is a local minimizer for \(f(x)\).