Maximize \( w = w(y) = b^T y \)
subject to \( A^T y \leq c, \quad y \geq 0 \).

Prove that if \( x \) and \( y \) are feasible solutions to the given problem and its dual, respectively, then \( w(y) \leq z(x) \). You should assume any properties of matrices that you need. Be sure to indicate how and where you make use of the facts that \( x \geq 0 \) and \( y \geq 0 \).

6. Show that \((-\sqrt{2/3}, \sqrt{1/6}, \sqrt{1/6})^T\) is a local minimizer for the problem

Minimize \( f(x) = f(x_1, x_2, x_3) = 3x_1 + x_2 + x_3 \)
subject to
\[
\begin{align*}
x_1 + x_2 + x_3 &= 0 \\
x_1^2 + x_2^2 + x_3^2 &= 1
\end{align*}
\]
by demonstrating that \((-\sqrt{2/3}, \sqrt{1/6}, \sqrt{1/6})^T\) satisfies the sufficient conditions presented in the course.

7. Consider the problem

Minimize \( f(x_1, x_2, x_3) = x_1^3 - x_1 x_2 + x_3^2 \)
subject to
\[
\begin{align*}
x_1 - x_3 &= 1 \\
2x_1 + x_2 &= 2
\end{align*}
\]
(a) Find a basis null space matrix \( Z \) for the constraint matrix \( A \) of this problem.
(b) For \( x = (1, -1, 1)^T \), find \( v \) and \( \lambda \) such that
\[
\nabla f(x) = Zv + A^T \lambda
\]

8. Show that \((0, -1)^T\) is a local minimizer for the problem

Minimize \( f(x) = f(x_1, x_2) = 2x_1^2 + x_2 \)
subject to
\[
\begin{align*}
x_2 &\geq x_1^2 - 1 \\
x_1 &\geq x_2
\end{align*}
\]