Maximize $w = w(y) = b^T y$
subject to $A^T y \leq c, \quad y \geq \mathbf{0}$.

Prove that if x and y are feasible solutions to the given problem and its dual, respectively, then $w(y) \leq z(x)$. You should assume any properites of matrices that you need. Be sure to indicate how and where you make use of the facts that $x \geq \mathbf{0}$ and $y \geq \mathbf{0}$.

6. Show that $(-\sqrt{2/3}, \sqrt{1/6}, \sqrt{1/6})^T$ is a local minimizer for the problem

Minimize $f(x) = f(x_1, x_2, x_3) = 3x_1 + x_2 + x_3$
subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1^2 + x_2^2 + x_3^2 &= 1 \end{aligned}$$

by demonstrating that $(-\sqrt{2/3}, \sqrt{1/6}, \sqrt{1/6})^T$ satisfies the sufficient conditions presented in the course.

7. Consider the problem

Minimize $f(x_1, x_2, x_3) = x_1^3 - x_1 x_2 + x_3^2$
subject to

$$\begin{aligned} x_1 \qquad - x_3 &= 1 \\ 2x_1 + x_2 \qquad &= 2 \end{aligned}$$

(a) Find a basis null space matrix Z for the constraint matrix A of this problem.
(b) For $x = (1, -1, 1)^T$, find v and λ such that

$$\nabla f(x) = Zv + A^T \lambda$$

8. Show that $(0, -1)^T$ is a local minimizer for the problem

Minimize $f(x) = f(x_1, x_2) = 2x_1^2 + x_2$
subject to

$$\begin{aligned} x_2 &\geq x_1^2 - 1 \\ x_1 &\geq x_2 \end{aligned}$$