1. Find all the local minimizers of the function

$$f(x_1, x_2) = (x_1 - \tfrac{1}{2}x_2)^2 + x_2^3 - 3x_2$$

and demonstrate that they satisfy the sufficient conditions presented in the class.

2. Show that $(-2/11, 6/11, -2/11)^T$ is a local minimizer for the problem

Minimize $f(x) = f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$
subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ -x_1 + 3x_2 - x_3 &= 2 \end{aligned}$$

by demonstrating that $(-2/11, 6/11, -2/11)^T$ satisfies the sufficient conditions presented in the course.

3. Using Newton's method to find the stationary points of

$$f(x) = f(x_1, x_2) = x_1^4 - 2x_1 - x_1^2 x_2 - x_2^3$$

we start with $\bar{x}_0 = (-1/2, 1/2)^T$. Calculate $\bar{x}_1$ in fraction form.

4. Show that $(1, 1)^T$ is a local minimizer for the problem

Minimize $f(x) = f(x_1, x_2) = x_1^2 + 2x_2^2 - x_1 x_2$
subject to

$$\begin{aligned} x_1 + x_2 &\geq 2 \\ x_2 &\geq x_1 \\ x_2 &\leq 2 \end{aligned}$$

by demonstrating that $(1, 1)^T$ satisfies the sufficient conditions presented in the course.

5. Consider the canonical form linear programming problem

Minimize $z = z(x) = c^T x$
subject to $Ax \geq b, \quad x \geq \mathbf{0}$

and its dual canonical form problem