Math 164: Homework #9, due on Friday, March 13

Please note, no late homework accepted.

[**1**] Solve the problem: Minimize $f(x_1, x_2) = \frac{1}{2}x_1^2 + x_2^2$ subject to

$$\begin{aligned} 2x_1 + x_2 &\geq 2 \\ x_1 - x_2 &\leq 1 \\ x_1 &\geq 0. \end{aligned}$$

[**2**] Let A be an $m \times n$ matrix whose rows are linearly independent. Prove that there exists a vector p such that $Ap = e_1$, where $e_1 = (1, 0, 0, ..., 0)^T$.

[**3**] Consider the bound-constrained problem
minimize $f(x)$
subject to $l \leq x \leq u$,
where l, u are vectors of lower and upper bounds, such that $l < u$.
Let x_* be a local minimizer. Show that:
if $x_{*,i} = l_i$, then $\frac{\partial f(x_*)}{\partial x_i} \geq 0$,
if $x_{*,i} = u_i$, then $\frac{\partial f(x_*)}{\partial x_i} \leq 0$,
if $l_i < x_{*,i} < u_i$, then $\frac{\partial f(x_*)}{\partial x_i} = 0$.

[**4**] Use the optimality conditions for nonlinear equality constraints to find all local solutions to the problem
minimize $f(x) = -x_1^2 + x_2^2$
subject to $2x_1^2 + x_2^2 = 4$.

[**5**] Use the optimality conditions for nonlinear inequality constraints to find all local solutions to the problem
minimize $f(x) = x_1 + x_2$
subject to $(x_1 - 1)^2 + x_2^2 \leq 2$
$(x_1 + 1)^2 + x_2^2 \geq 2$.