**Math 164: Homework #8, due on Friday, March 6**

- Reading: Sections 14.2 and 14.3.

**[1]** Consider the problem

$$\text{minimize } f(x) = x_1^2 + x_1^2 x_3^2 + 2x_1x_2 + x_2^4 + 8x_2$$

$$\text{subject to } 2x_1 + 5x_2 + x_3 = 3.$$

(a) Determine which of the following points are stationary points:
(i) $(0,0,2)^T$; (ii) $(0,0,3)^T$; (iii) $(1,0,1)^T$
(b) Determine whether each stationary point is a local minimizer, a local maximizer or a saddle point.

**[2]** Consider the problem of finding the minimum distance from a point $r$ to a set $\{x : a^T x = b\}$. The problem can be written as:

minimize $\quad f(x) = \frac{1}{2}(x-r)^T(x-r)$,
subject to $\quad a^T x = b$.

Prove that the solution is given by $x_* = r + \frac{b - a^T r}{a^T a} a$.

**[3]** Solve the problem

$$\text{maximize } f(x) = x_1 x_2 x_3$$

$$\text{subject to } \frac{x_1}{a_1} + \frac{x_2}{a_2} + \frac{x_3}{a_3} = 1 \quad (a_1, a_2, a_3 > 0)$$

**[4]** Consider the problem

minimize $\quad f(x) = 3x_1^2 - \frac{1}{2}x_2^2 - \frac{1}{2}x_3^2 + x_1x_2 - x_1x_3 + 2x_2x_3$
subject to $\quad 2x_1 - x_2 + x_3 = 2$.

(a) Solve this problem. Use the Lagrange multiplier to estimate the minimum value of $f$ under the perturbed constraint $2x_1 - x_2 + x_3 = 2 + \delta$. Compare the estimated minimum objective value to the actual minimum for $\delta = 0.25$.

(b) Determine the exact solution to the perturbed problem as a function of $\delta$. Are there any limits on how large $\delta$ can be before this result ceases to be valid ?