Math 164: Homework #7, due on Friday, February 27

• Please review Section 2.3.1, Appendices A.7.1, B4, B5, and Section 2.6.

[**1**] Determine if

$$f(x_1, x_2) = 2x_1^2 - 3x_1x_2 + 5x_2^2 - 2x_1 + 6x_2$$

is convex, concave, both, or neither for $x \in R^2$. (Review Section 2.3.1).

[**2**] Find the first 3 terms of the Taylor series for

$$f(x_1, x_2) = 3x_1^4 - 2x_1^3x_2 - 4x_1^2x_2^2 + 5x_1x_2^3 + 2x_2^4$$

at the point $x_0 = (1, -1)^T$. Evaluate the series for $p = (.1, .01)^T$ and compare with the actual value of $f(x_0 + p)$. (Review Section 2.6).

[**3**] Consider the following function

$$f(x) = 15 - 12x - 25x^2 + 2x^3.$$

(a) Use the first and second order derivatives to find the local maxima and local minima of f.

(b) Show that f has neither a global maximum nor a global minimum.

[**4**] Consider the function

$$f(x) = 8x_1^2 + 3x_1x_2 + 7x_2^2 - 25x_1 + 31x_2 - 29.$$

Find all stationary points of this function, and determine whether they are local minimizers and maximizers. Does this function have a global minimizer or a global maximizer ?

[**5**] Consider the problem

$$\min_x \|Ax - b\|_2^2,$$

where A is an $m \times n$ matrix with $m \geq n$, and b is a vector of length m. Assume that the rank of A is equal to n.

(a) Write down the first-order necessary condition for optimality. Is this also a sufficient condition ?

(b) Write down the optimal solution in closed form.

[**6**] Use Newton's method to find an approximate solution of
minimize

$$f(x_1, x_2) = 5x_1^4 + 6x_2^4 - 6x_1^2 + 2x_1x_2 + 5x_2^2 + 15x_1 - 7x_2 + 13.$$

Use the initial guess $(1, 1)^T$. Make sure that you have found a minimum and not a maximum.

[**7**] (Read Appendix B5) Consider the problem

$$\text{minimize } f(x) = \frac{1}{2}x^TQx - c^Tx.$$

(a) Write the first-order necessary condition. When does a stationary point of f exist ?

(b) Under what conditions on Q does a local minimizer exist ?

(c) If Q is a positive definite matrix, prove that Newton's method will determine the minimizer of f in one iteration, regardless of the starting point.