Math 164: Homework #5, due on Friday, February 13

[**1**] Suppose that a linear program in standard form, with bounded feasible region, has l optimal extreme points $\{v_1, v_2, ..., v_l\}$. Prove that a point is optimal for the linear program if, and only if, it can be expressed as a convex combination of $\{v_1, v_2, ..., v_l\}$.

[**2**] Solve the following linear program using the simplex method (graph the feasible region, and outline the progress of the algorithm).

$$\text{maximize } z = 7x_1 + 8x_2, \text{ subject to } \begin{cases} 4x_1 + x_2 \le 100 \\ x_1 + x_2 \le 80 \\ x_1 \le 40 \\ x_1, x_2 \ge 0. \end{cases}$$

[**3**] Compute a basis matrix for the null space of the matrix A and express the point x as $x = p + q$, where p is in the null space of A and q is in the range space of A^T:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 2 \end{pmatrix}.$$

[**4**] Consider the linear program: Minimize $z = x_1 - x_2$ subject to
$-x_1 + x_2 \le 1$
$x_1 - 2x_2 \le 2$
$x_1, \; x_2 \ge 0.$
Derive an expression for the set of optimal solutions to this problem, and show that this set is unbounded.

[**5**] Find the dual of
minimize $z = 3x_1 - 5x_2 - 7x_3$,
subject to
$x_1 + 5x_2 - 8x_3 \ge 5$
$4x_1 - 2x_2 + 7x_3 \ge 7$
$x_1, x_2, x_3 \ge 0.$