Math 164: Homework #3, due on Friday, January 30

Please note: exceptionally there will be no office hours with the instructor the week of Jan. 26. Please email if you have questions or see the T.A.

[**1**] Prove that a function f is concave if and only if $-f$ is convex.

[**2**] Let f be a convex function on the convex set S of R^n. Let k be a nonzero scalar, and define $g(x) = kf(x)$. Prove that if $k > 0$ then g is a convex function on S, and if $k < 0$ then g is a concave function of S.

[**3**] Consider a feasible region S defined by a set of linear constraints

$$S = \{x : \ Ax \le b\},$$

where A is an $m \times n$ matrix and b is a column vector.

(a) Prove that S is convex.

(b) Derive the conditions that must be satisfied by a feasible direction p at a point $x \in S$.

[**4**] Consider the problem

$$\text{minimize } f(x)$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 = 6, \ x_1 \ge 0, \ x_2 \ge 0, \ x_3 \ge 0.$$

(a) Find the sets of all feasible directions at points $x_a = (0,0,2)^T$, $x_b = (3,0,1)^T$, and $x_c = (1,1,1)^T$.

(b) Using (a), verify that $p = (3,0,-1)^T$ is a feasible direction for $x_c = (1,1,1)^T$; then find an upper bound on the step length α so that $x_c + \alpha p$ is a feasible point, with $p = (3,0,-1)^T$.

[**5**] Solve the following linear program graphically:

$$\text{maximize } z = 6x_1 - 3x_2, \text{ subject to } \begin{cases} 2x_1 + 5x_2 \ge 10, \\ 3x_1 + 2x_2 \le 40, \\ x_1, x_2 \le 15. \end{cases}$$