Math 164: Homework #2, due on Friday, January 23

Please solve as many problems as you can from the textbook. Also, please review the material in the Appendices, at the end of the textbook, and read Sections 2.2. and 2.3 from the textbook.

[**1**] Consider the feasible region defined by the constraints

$$1-(x_1)^2-(x_2)^2 \geq 0, \quad 1-x_1-x_2 \geq 0, \text{ and } x_2 \geq 0.$$

For each of the following points, determine whether the point is feasible or infeasible, and (if it is feasible) whether it is interior to or on the boundary of each of the constraints:

$$x_a = (\frac{1}{2}, \frac{1}{2})^T, \ x_b = (1,1)^T, \ x_c = (-1,0)^T, \ x_d = (-\frac{1}{2}, 0)^T, \ x_e = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})^T.$$

[**2**] Consider the problem

$$\begin{array}{lll} \text{minimize} & x_1 & \\ \text{subject to} & (x_1)^2+(x_2)^2 & \leq 4, \\ & (x_1)^2 & \geq 1. \end{array}$$

Graph the feasible set. Use the graph to find all local minimizers for the problem, and determine which of those are also global minimizers.

[**3**] Let $S_1 = \{x = (x_1, x_2) : \ x_1 + x_2 \leq 1, \ x_1 \geq 0\}$, and $S_2 = \{x = (x_1, x_2) : x_1 - x_2 \geq 0, \ x_1 \leq 1\}$, and let $S = S_1 \cup S_2$. Prove that S_1 and S_2 are both convex sets, but that S is not a convex set.

(this shows that the union of convex sets is not necessarily convex, but the intersection is, see exercise 1 page 24).

[**4**] Let $f : R^n \to R$ be a convex function, and let $g : R \to R$ be a convex nondecreasing function. Prove that the composite function $h : R^n \to R$ defined by $h(x) = g(f(x))$ is convex.

[**5**] Consider the one-variable function

$$f(x) = (x+1)x(x-2)(x-5) = x^4 - 6x^3 + 3x^2 + 10x.$$

Graph this function and locate (approximately) the stationary points, local minima, and global minima.