**Math 164: Homework #1, due on Friday, January 16**

Reading: Chapter 1.

[**1**] (Fitting a quadratic function to data) The following points in the plane are assumed to lie on the graph of a quadratic function $(t, b(t))$. These points, denoted by $(t_i, b_i)$, have the coordinates $(0, 1)$, $(2, 7)$, and $(5, 46)$. Find the quadratic function and plot its graph.

[**2**] A manufacturer of office furniture is trying to maximize the monthly revenue of the factory. Various orders have come in that the company could accept. They include desks, bookshelves, cabinets with doors, and cabinets with drawers. The table below indicates the quantities of materials and labor required to assemble the four types of furniture, as well as the revenue earned. Suppose that 6000 units of wood and 2000 units of labor are available. Formulate the linear programming model that will maximize the revenue under the given conditions, where $x_i$, $i = 1, 2, 3, 4$ is the number of pieces of furniture to be produced for each type (see Section 1.4)

| Piece | Labor | Wood | Revenue |
|---|---|---|---|
| desk | 8 | 12 | 200 |
| bookshelf | 6 | 10 | 100 |
| cabinet with doors | 2 | 25 | 150 |
| cabinet with drawers | 4 | 20 | 200 |

[**3**] Four buildings are to be connected by electrical wires. The positions of the buildings are as follows: the first building's shape is an ellipse with center $(0, 0)$ and horizontal and vertical axes 0.5 and 0.6. The second building is a disk with center $(4, 0)$ and radius 1. The other two buildings are squares centered at $(0, 4)$ and at $(4, 4)$, with sides parallel with the axes and of length 2. The electrical wires will be joined at some central point $(x_0, y_0)$, and will connect to building $i$ at position $(x_i, y_i)$.

(a) Plot the positions of the four buildings in the plane.

(b) Formulate the non-linear optimization problem that minimizes the amount of wire used (see Section 1.6)

[**4**] Assume that $m$ points $(t_i, b_i)$ are given in the plane, $i = 1, 2, ..., m$. Formulate a least-squares minimization problem in the unknown $x = (x_1, x_2, x_3, x_4, x_5)$, for fitting a curve defined by $b(t) = x_1 + x_2 e^{x_3 t} + x_4 e^{x_5 t}$ through the points in an optimal way (see Section 1.5)

[**5**] (Minimum distance from a point to a set). In each case, formulate a (nonlinear) optimization problem that:

(i) Finds the minimum distance from a given point $r$ to a hyperplane $S = \{x : \ a^t x = b\}$.

(ii) Finds the minimum distance from a given point $r$ to the unit sphere.

(see Section 1.6).