b) We know that a vector d is a direction of unboundedness for the pb. $\begin{cases} Ax=b \\ x\geq 0 \end{cases}$ if

$d \geq 0$ and $A\cdot d = 0$

We can find A to be

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 & 0 \\ 3 & 0 & -2 & 0 & 1 \end{pmatrix}$$

$$A\cdot d = \begin{pmatrix} 2 & -1 & 0 & 1 & 0 \\ 3 & 0 & -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 4 \\ 1 \\ 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 2-3+1 \\ 3-8+5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

c) Using (a) and (b) we have $x+\gamma d =$
$= (2,1,3,0,2)^T + \gamma\cdot(1,3,4,1,5)^T = (2+\gamma, 1+3\gamma, 3+4\gamma, \gamma, 2+5\gamma)$

We are imposing the condition

$$3(2+\gamma) - 2(3+4\gamma) = -100$$
$$6+3\gamma - 6 - 8\gamma = -100$$
$$-5\gamma = -100 \Rightarrow \boxed{\gamma = 20}.$$

d) ~~We observe that~~ Since ~~d~~ $(1,3,4,1,5)^T$ is a direction of unboundedness, then we have that ~~$x+\gamma d$~~ $= (2,1,3,0,2)^T + \gamma\cdot(1,3,4,1,5)^T$ $= (2+\gamma, 1+3\gamma, 3+4\gamma, \gamma, 2+5\gamma)$ is a feasible solution, for every γ.