5. Consider the canonical form linear programming problem

Minimize $z = z(x) = c^T x$
subject to $Ax \geq b, \quad x \geq \mathbf{0}$

and its dual canonical form problem

Maximize $w = w(y) = b^T y$
subject to $A^T y \leq c, \quad y \geq \mathbf{0}.$

Prove that if x and y are feasible solutions to the given problem and its dual, respectively, then $w(y) \leq z(x)$. You should assume any properites of matrices that you need. Be sure to indicate how and where you make use of the facts that $x \geq \mathbf{0}$ and $y \geq \mathbf{0}$.

$A^T y \leq c$ then $(A^T y)^T \leq c^T$, that is
$y^T A \leq c^T$ so $y^T A x \leq c^T x$ since $x \geq \underline{0}$
$Ax \geq b$ then $x^T A^T \geq b^T$ so $x^T A^T y \geq b^T y$
since $y \geq \underline{0}$ Now $x^T A^T y$ is one-by-one
so $x^T A^T y = (x^T A^T y)^T = y^T A x$ and thus
$w(y) = b^T y \leq x^T A^T y = y^T A x \leq c^T x = z(x)$