Math 164: Homework #8, due on Wednesday, May 27

[**1**] (read Appendix B5) Consider the problem

$$\text{minimize } f(x) = \frac{1}{2}x^T Q x - c^T x.$$

(a) Write the first-order necessary condition. When does a stationary point of f exist ?
(b) Under what conditions on Q does a local minimizer exist ?
(c) If Q is a positive definite matrix, prove that Newton's method will determine the minimizer of f in one iteration, regardless of the starting point.

[**2**] Consider the problem

$$\min_x \|Ax - b\|_2^2,$$

where A is an $m \times n$ matrix with $m \geq n$, and b is a vector of length m. Assume that the rank of A is equal to n.
(a) Write down the first-order necessary condition for optimality. Is this also a sufficient condition ?
(b) Write down the optimal solution in closed form.

[**3**] Compute a basis matrix for the null space of the matrix A and express the point x as $x = p + q$, where p is in the null space of A and q is in the range space of A^T:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 2 \end{pmatrix}.$$

[**4**] Consider the problem

$$\text{minimize } f(x) = x_1^2 + x_1^2 x_3^2 + 2x_1 x_2 + x_2^4 + 8x_2$$

$$\text{subject to } 2x_1 + 5x_2 + x_3 = 3.$$

(a) Determine which of the following points are stationary points:
(i) $(0,0,2)^T$; (ii) $(0,0,3)^T$; (iii) $(1,0,1)^T$
(b) Determine whether each stationary point is a local minimizer, a local maximizer or a saddle point.

[**5**] Solve the problem

$$\text{maximize } f(x) = x_1 x_2 x_3$$

$$\text{subject to } \frac{x_1}{a_1} + \frac{x_2}{a_2} + \frac{x_3}{a_3} = 1 \quad (a_1, a_2, a_3 > 0)$$