

Math 164, Vese: Homework #6, due on Wednesday, May 13

- Midterm exam on Monday, May 11. Sample midterm problems are posted. Sections covered: 1, 2.2-2.4, 3.1, 4.1-4.4, 5.2, 6.1-6.2
- Exceptionally, there are no office hours with the instructor on Monday, May 4.
- Office hours before midterm: Wednesday 1-2pm. Friday, 1-3pm, or by appointment.

[1] Find the dual of

$$\begin{aligned} &\text{minimize } z = 3x_1 - 5x_2 - 7x_3, \\ &\text{subject to} \\ &x_1 + 5x_2 - 8x_3 \geq 5 \\ &4x_1 - 2x_2 + 7x_3 \geq 7 \\ &x_1, x_2, x_3 \geq 0. \end{aligned}$$

[2] Find the dual of

$$\begin{aligned} &\text{maximize } z = 6x_1 - 3x_2 - 2x_3 + 5x_4, \\ &\text{subject to} \\ &4x_1 + 3x_2 - 8x_3 + 7x_4 = 11 \\ &3x_1 + 2x_2 + 7x_3 + 6x_4 \geq 23 \\ &7x_1 + 4x_2 + 3x_3 + 2x_4 \leq 12 \\ &x_1, x_2 \geq 0, x_3 \leq 0, x_4 \text{ free} \end{aligned}$$

Verify that the dual of the dual is the primal.

[3] Find the dual to the problem

$$\begin{aligned} &\text{minimize } z = c^T x, \\ &\text{subject to } b_1 \leq Ax \leq b_2, \\ &x \geq 0. \end{aligned}$$

[4] Consider the linear program

$$\text{maximize } z = -x_1 - x_2, \text{ subject to } \begin{cases} -x_1 + x_2 \geq 1 \\ 2x_1 - x_2 \leq 2 \\ x_1, x_2 \geq 0. \end{cases}$$

Find the dual to the problem. Solve the primal and the dual graphically, and verify that the results of the strong duality theorem hold.

[5] Consider the linear program

$$\begin{aligned} &\text{minimize } z = 2x_1 + 9x_2 + 3x_3 \\ &\text{subject to } -2x_1 + 2x_2 + x_3 \geq 1 \\ &x_1 + 4x_2 - x_3 \geq 1 \\ &x_1, x_2, x_3 \geq 0. \end{aligned}$$

- Find the dual to this problem and solve the dual problem graphically.
- Use complementarity slackness to obtain the solution to the primal.

[6] Consider the primal linear programming problem

$$\begin{aligned} &\text{Minimize } z = c^T x \\ &\text{subject to } Ax \leq b, \\ &x \geq 0. \end{aligned}$$

Assume that this problem and its dual are both feasible. Let x_* be an optimal solution vector to the primal, let z_* be its associated objective value, and let y_* be an optimal solution vector to the dual problem. Show that $z_* = y_*^T Ax_*$.